THE DESIGN AND MEASUREMENT OF A MICROWAVE MODE TRANSDUCER

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ABSTRACT

A mode transducer to separate the TE_{10} and TE_{20} modes propagating in multimode waveguide and couple the power from each into single mode waveguides is described. The construction of an experimental model is shown and data on its performance given. The capabilities of this type of mode transducer are discussed and modifications to give improved performance are suggested.

The writer wishes to express his appreciation to Professor Roy M. Johnson for suggesting the study and for providing, along with Captain Ralph Lary, USMC, assistance and encouragement with this project.

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1. Introduction

Investigation of the scattering by a slot in a multimode waveguide required the development of a specialized microwave bridge[1] for taking experimental data. The requirement for a better mode transducer for use in this bridge initiated the project described in this paper.

When two or more modes may propagate in the same waveguide, it is not easy to determine by measurement the amplitude and phase of each. The difficulties are mainly practical ones, which would not arise if the experimenter had at his disposal infinitesimal probes that could be moved about within the guide at will without disturbing the existing fields.

The slotted-line technique is not applicable for it is impossible to cut a slot in the multimode guide that will be everywhere parallel to the lines of current flow in each of the two modes and will therefore seriously disturb the field of neither.

The method of measuring the fields in the multimode waveguide that was adopted consisted of first transforming the power in the TE_{20} mode from the multimode waveguide to TE_{10} in single mode guide and similarly transforming the TE_{10} power in the multimode guide to TE_{10} in another single mode guide. The phase and amplitude of the fields

[1] Capt. Ralph L. Lary, USMC, A Microwave Bridge for the Accurate Measurement of Scattering Matrices Generated by Obstacles in Multimode Waveguide, U.S.N.P.S. Thesis, 1862

propagating in the single mode waveguides can now be measured and serve to indicate the fields existing in the multimode waveguide. The device used to affect this transormation is a mode transducer.

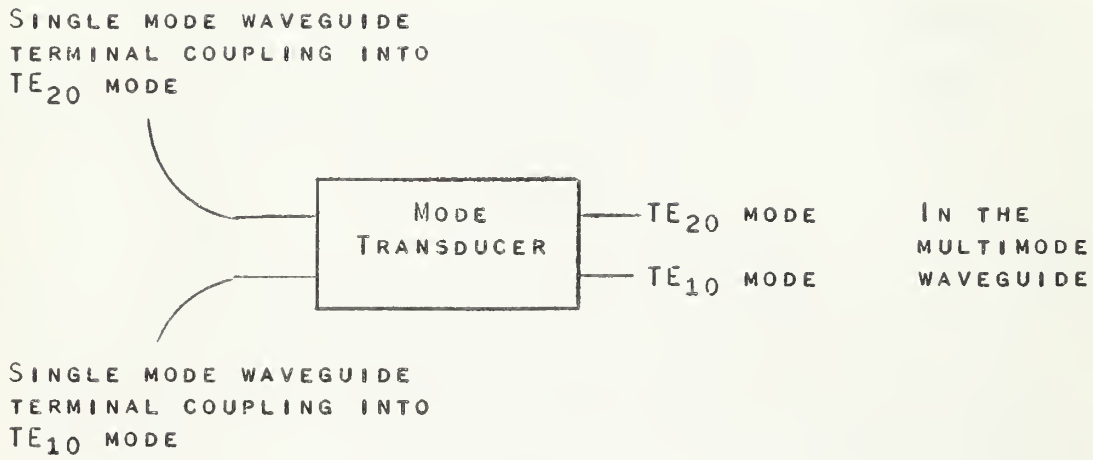


Figure 1

An ideal mode transducer will be defined to have the following properties:

A. It should present a perfect match to power entering either of the single mode ports when the multimode port is terminated in a multimode matched load (matched for both modes). Also there should be no reflection of either the TE_{10} or the TE_{20} energy entering the multimode port when the single mode ports are terminated in matched loads.

B. There should be no coupling between modes. TE_{10} entering the multimode port should not be reflected as TE_{20} out of the multimode port or transmitted through the TE_{20} single mode port. TE_{20} entering the multimode port

should not be reflected as TE_{10} out of the multimode port or transmitted through the TE_{10} single mode port. Similarly energy entering either of the single mode ports should be converted entirely to the desired mode in the multimode waveguide.

The performance of a microwave mode transducer can be specified by the degree to which these two parameters, match and isolation, are obtained.

2. Representation of a Mode Transducer by the Scattering Matrix

A mode transducer can be viewed as a junction of four waveguides. Two of the junctions are the single mode waveguide ports and the other two ports represent the two propagating modes possible in the multimode waveguide. Figure 2 depicts the mode transducer as a junction of four waveguides with only one mode propagating in each arm.

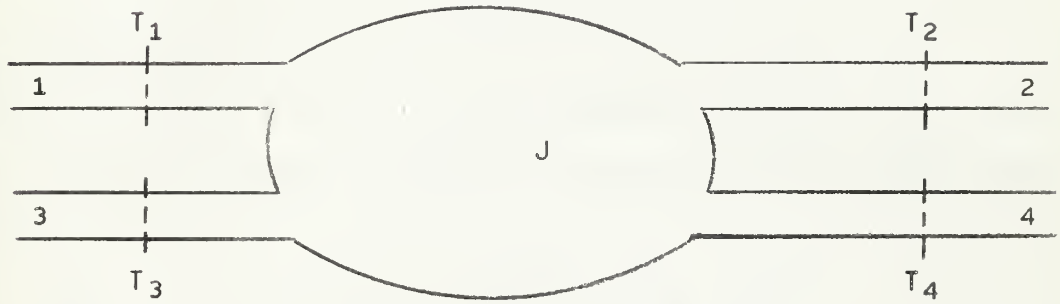


Figure 2

Terminal number one represents the single mode waveguide port that couples into the TE_{20} mode in the multimode waveguide.

Terminal number two represents the TE_{20} mode in the multimode guide.

Terminal number three represents the single mode waveguide port that couples into the TE_{10} mode in the multimode guide.

Terminal number four represents the TE_{10} mode in the multimode guide.

A COMPLETE DESCRIPTION OF THE FIELDS WITHIN THE DISCONTINUITY REGION OF THE JUNCTION, J, WOULD REQUIRE, IN ADDITION TO THE DOMINANT PROPAGATING MODES, AN INFINITY OF NON-PROPAGATING MODES. THIS COMPLICATION IN THE FIELD DESCRIPTION THAT EXISTS AT THE JUNCTION CAN BE AVOIDED BY LOOKING ONLY AT THE DOMINANT MODES AT TERMINAL PLANES T_1 THROUGH T_4 LOCATED SUFFICIENTLY DISTANT FROM THE JUNCTION SO THAT THE NON-PROPAGATING HIGHER-ORDER MODES HAVE VANISHED. IN THE SCATTERING TYPE OF REPRESENTATION THE FIELDS ARE DESCRIBED BY THE AMPLITUDES OF THE INCIDENT AND REFLECTED WAVES AT EACH OF THE TERMINAL PLANES. FOR A TE MODE THE FIELDS AT THE TERMINAL PLANE T_J $(Z=Z_J)$ ARE GIVEN BY:/1/

$$\vec{E}_J(X,Y,Z_J) = A_J \vec{\mathcal{E}}_J^{+} + B_J \vec{\mathcal{E}}_J^{-}$$

$$\vec{H}_J(X,Y,Z_J) = A_J \vec{\mathcal{H}}_J^{+} + B_J \vec{\mathcal{H}}_J^{-}$$

WHERE $\vec{\mathcal{E}}_J$ AND $\vec{\mathcal{H}}_J$ ARE VECTOR MODE FUNCTIONS CHARACTERISTIC OF THE TRANSVERSE FORM OF THE DESIRED MODE IN THE J^{TH} GUIDE.

FOR TE_{MN} MODES IN RECTANGULAR GUIDE:/2/

$$\vec{\mathcal{E}} = \left(E_X \vec{I}_X + E_Y \vec{I}_Y \right) e^{j\omega t}$$

$$\vec{\mathcal{H}} = \left(H_X \vec{I}_X + H_Y \vec{I}_Y + H_Z \vec{I}_Z \right) e^{j\omega t}$$

WHERE:

$$E_X = \frac{j\omega \mu K_Y}{K_X^2 + K_Y^2} \cos K_X X \ \sin K_Y Y$$

$$E_y = -\frac{j\omega\mu K_x}{K_x^2 + K_y^2} \sin K_x x \cos K_y y$$

$$H_x = \frac{\gamma_{mn} K_x}{K_x^2 + K_y^2} \sin K_x x \cos K_y y$$

$$H_y = \frac{\gamma_{mn} K_y}{K_x^2 + K_y^2} \cos K_x x \sin K_y y$$

$$H_z = \cos K_x x \cos K_y y$$

$$\gamma_{mn} = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2 \mu \epsilon}$$

$$K_x = \frac{m\pi}{a} \qquad K_y = \frac{n\pi}{b}$$

THE PLUS AND MINUS SUPERSCRIPT INDICATE PLUS AND MINUS TRAVELING WAVES. A_J AND B_J ARE, RESPECTIVELY, THE COMPLEX AMPLITUDES OF THE ELECTRIC FIELD IN THE INCIDENT AND REFLECTED WAVE COMPONENTS OF THE DESIRED MODE FIELD AT TERMINAL T_J. SINCE $\vec{\mathcal{E}}_{m,n}$ AND $\vec{\mathcal{H}}_{m,n}$ ARE BASIS VECTORS, ANY SOLUTION TO THE FIELD EQUATIONS IN RECTANGULAR GUIDE MAY BE REPRESENTED BY SUITABLE FOURIER METHODS.

THE FUNDAMENTAL EXISTENCE THEOREM /1/ APPLICABLE TO THE SCATTERING FORMULATION OF A FIELD PROBLEM STATES THAT THE AMPLITUDES OF THE SCATTERED WAVES AT THE VARIOUS TERMINALS ARE UNIQUELY RELATED TO THE AMPLITUDES OF THE INCIDENT WAVES THEREON. IN THE CASE OF LINEAR OBSTACLES, THE AMPLITUDES OF THE REFLECTED WAVES SET UP AT THE TERMINAL

PLANES $T_1, \ldots T_J$ BY A SINGLE INCIDENT WAVE A_1 AT T_1, OR A_2 AT $T_2, \ldots$, ETC., ARE:

$$
\begin{array}{lll}
B_1 = S_{11}A_1 & B_1 = S_{12}A_2 & B_1 = S_{1J}A_J \\
B_2 = S_{21}A_1 & B_2 = S_{22}A_2 & B_2 = S_{2J}A_J \\
\cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot \\
B_J = S_{J1}A_1 & B_J = S_{J2}A_2 & B_J = S_{JJ}A_J
\end{array}
$$

BY SUPERPOSITION THE AMPLITUDES OF THE SCATTERED WAVES RESULTING FROM THE SIMULTANEOUS INCIDENCE OF WAVES OF AMPLITUDES $A_1, \ldots A_J$ ARE:

$$
\begin{array}{l}
B_1 = S_{11}A_1 + S_{12}A_2 \ldots + S_{1J}A_J \\
B_2 = S_{21}A_1 + S_{22}A_2 \ldots + S_{2J}A_J \\
\cdot \\
\cdot \\
B_J = S_{J1}A_1 + S_{J2}A_2 \ldots + S_{JJ}A_J
\end{array}
$$

WHERE THE SCATTERING COEFFICIENT S_{JK} IS A MEASURE OF THE AMPLITUDE AND PHASE OF THE WAVE SCATTERED INTO THE J^{TH} GUIDE BY A WAVE OF UNIT AMPLITUDE INCIDENT INTO THE K^{TH} GUIDE. NOTE THEREFORE THAT S_{JJ} REPRESENTS THE REFLECTION COEFFICIENT AT THE TERMINAL T_J WHEN ALL OTHER TERMINALS ARE MATCHED. IN MATRIX FORM THE SCATTERING PROPERTIES OF A WAVEGUIDE STRUCTURE ARE EXPRESSED BY THE ARRAY OF COEFFICIENTS CALLED THE SCATTERING MATRIX.

$$S = \begin{pmatrix} S_{11} & S_{12} & \cdots & S_{1N} \\ S_{21} & & & \\ \cdot & & & \\ \cdot & & & \\ S_{N1} & & & S_{NN} \end{pmatrix}$$

IT CAN BE SHOWN /1/ THAT THE SCATTERING MATRIX IS SYMMETRICAL AND FOR NON-DISSIPATIVE STRUCTURES IT IS UNITARY. THIS MEANS THAT:

$$S_{JK} = S_{KJ}$$

AND

$$\sum_{\alpha=1}^{N} S^{*}_{\alpha m} S_{\alpha n} = \delta_{mn} = \begin{cases} 1 & \text{IF } m = n \\ 0 & \text{IF } m \neq n \end{cases}$$

FOR A FOUR TERMINAL JUNCTION IT HAS BEEN SHOWN BY W. H. KUMMER /3/ THAT THE FOLLOWING RELATIONS HOLD:

$S_{13} = S_{14} = S_{23} = S_{24}$, $S_{12} = 1 - S_{11}$, $S_{11} = S_{22}$.

APPLYING THESE RELATIONSHIPS AND THE RECIPROCITY RELATIONS TO THE SCATTERING MATRIX FOR A FOUR TERMINAL MODE TRANSDUCER IT CAN BE SEEN THAT ONLY THREE SCATTERING COEFFICIENTS ARE REQUIRED TO COMPLETELY SPECIFY THE SCATTERING MATRIX.

$\longrightarrow S_{11} = S_{22}$

$\longrightarrow S_{33} = S_{44}$

$\longrightarrow S_{13} = S_{31} = S_{14} = S_{41} = S_{23} = S_{32} = S_{24} = S_{42}$

$S_{12} = 1 - S_{11} = S_{21}$

$S_{34} = 1 - S_{33} = S_{43}$

The scattering coefficients of an ideal mode transducer would be:

$$S_{12} = S_{21} = S_{34} = S_{43} = 1$$

$$S_{11} = S_{13} = S_{14} = 0$$

$$S_{22} = S_{23} = S_{24} = 0$$

$$S_{31} = S_{32} = S_{33} = 0$$

$$S_{41} = S_{42} = S_{44} = 0$$

The scattering matrix of an ideal mode transducer would be:

$$S = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

3. Design of the Mode Transducer

The transition from the TE_{10} mode in the multimode guide to the TE_{10} mode single mode guide is a relatively simple matter. Since the field configuration is the same in both guides the problem is almost equivalent to that of providing a reflectionless junction between two transmission lines of different characteristic impedance. A quarter wave matching section is a common method of providing a reflectionless junction but it has the disadvantage of being frequency sensitive. In order to allow operation over a broad band of frequencies the impedance transformation was made by means of an intermediate section in which a gradual change from the dimensions of one guide to those of the other takes place (see Figure 3). Provided this tapered section is at least a few wavelengths long there will be very little reflection over a wide frequency band. Since the dimensions of the single mode guide are such that the TE_{20} mode cannot propagate freely, any TE_{20} energy incident will see a reactive termination and will be attenuated rapidly in the single mode guide.

Figure 3

The transition from TE_{20} in multimode guide to TE_{10} in single mode guide can be accomplished in various ways /2/. The method used was chosen because "mode purity", that is, the freedom of excitation of the undesired TE_{10} mode, depends on physical symmetry only, and is, ideally, not a function of frequency. Figure 4 shows how this was accomplished. The septum at A divides the fields midway across the multimode guide and they propagate as TE_{10} in arms B and C. As shown by the E field vectors, TE_{20} incident upon the junction becomes TE_{10} with identical amplitudes in each arm but 180° out of phase. The 90° twists, clockwise in one arm and counterclockwise in the other, give a net 180° rotation between the two arms bringing the E fields back in phase at junction D where they add and emerge at E as a TE_{10} field in single mode waveguide. Most of the TE_{10} incident at the multimode port is shorted by the septum at A but the remainder becomes TE_{10} but with the same phase in each arm. After being rotated at B and C the net result is a 180° phase difference between the fields and complete cancellation at the junction D. If the cross section of the guide is changed slowly and uniformly over a distance of several wave lengths the TE_{20} incident at the multimode port will be transformed into TE_{10} in the single mode guide with little reflection. The degree of mode purity that can be obtained depends upon how closely the electrical lengths of the two arms can be matched and how accurately the fields split at

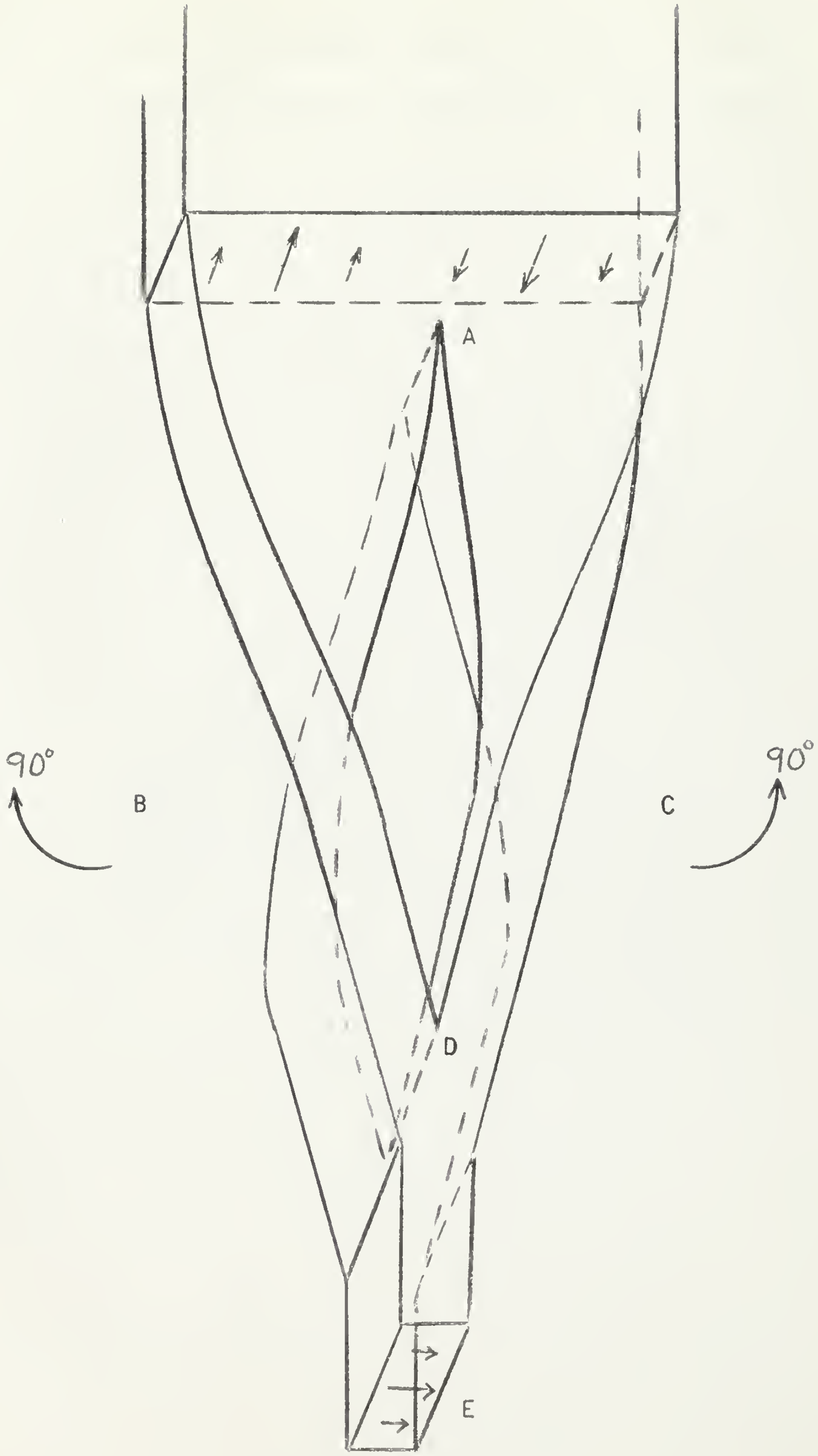


FIGURE 4

JUNCTION D. IF THIS TRANSDUCER COULD BE CONSTRUCTED WITH MIRROR SYMMETRY ABOUT THE CENTER LINE COMPLETE ISOLATION BETWEEN THE TE_{10} AND TE_{20} MODES WOULD BE OBTAINED AND, FURTHER, THIS ISOLATION WOULD BE FREQUENCY INDEPENDENT.

4. Construction of the Mode Transducer

The design principles just described were applied to the construction of a transducer for separating the TE_{20} and TE_{10} in multimode guide and transforming them to single mode guide. Since both of the methods used to transform the fields required access to the full width of the multimode guide, another septum was introduced splitting the narrow dimension of the guide, and consequently dividing the E field equally above and below the septum (Figure 5).

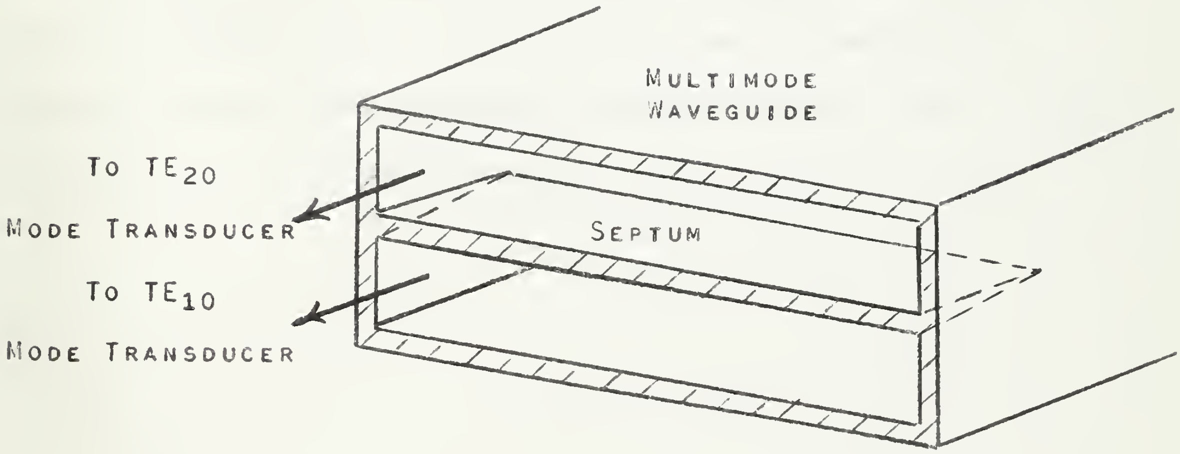


Figure 5

The lower section of the guide tapered uniformly into a cross section of single mode dimensions thereby forming the TE_{10} port. The upper section of the multimode guide was

bifurcated and the cross sections of each arm tapered to conform to the dimensions of X band single mode guide. At these points two pieces of waveguide, with opposite 90° twists, were attached. After the twists the guides came together again in a tapered E plane Y junction that ended in a standard X band waveguide flange forming the TE_{20} port. At the E plane Y junction a bendable septum extending into the common arm was provided to allow for adjusting the division of the E field at that point. Details of the construction can be seen in Figures 6, 7, and 8. The three pieces in Figure 6 were milled out of brass stock. The twists in Figure 7 were fabricated from standard RG58 X band waveguide and the Y junction was made by sandwiching together two beveled sections of waveguide and a sheet brass septum.

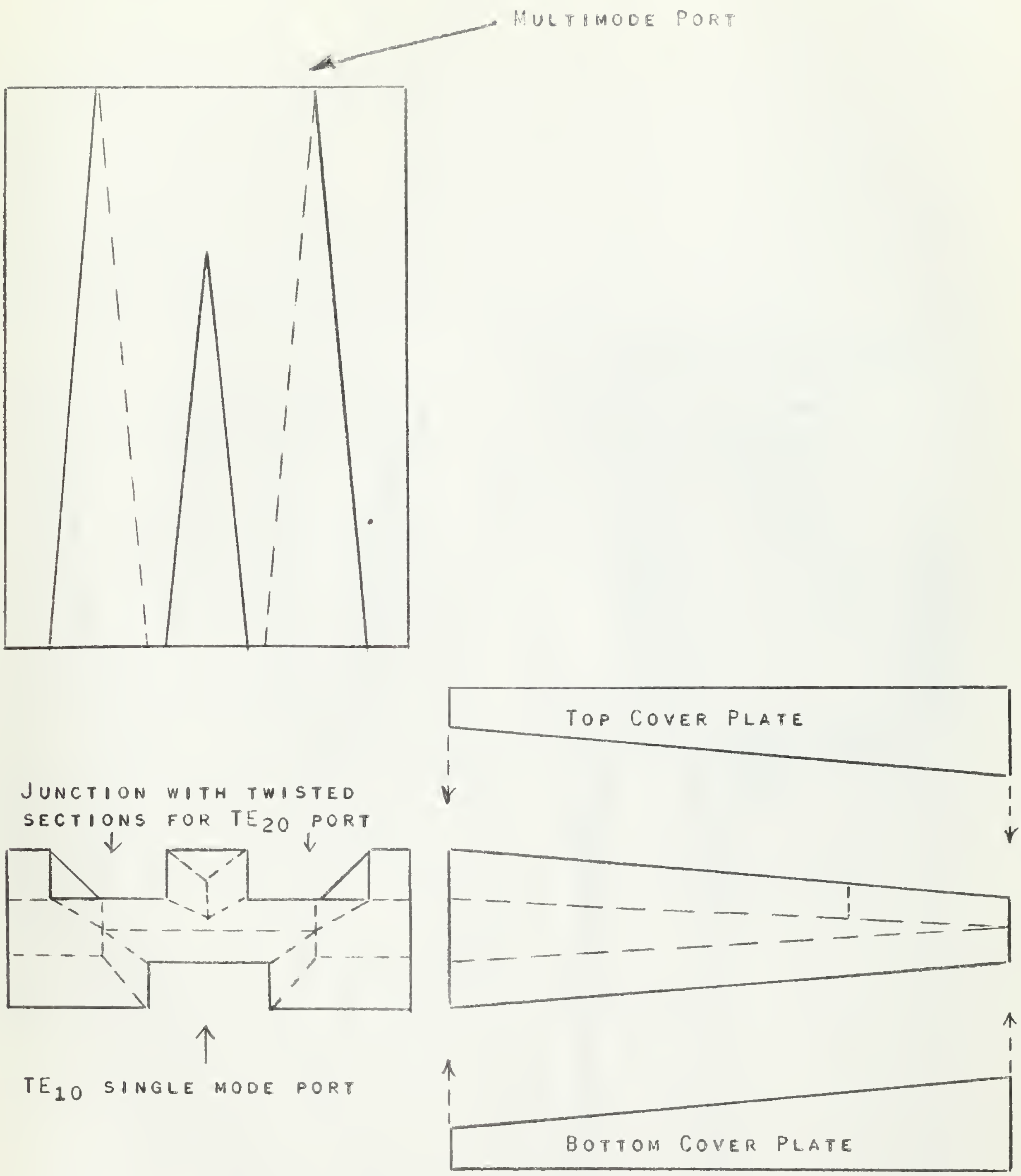

MULTIMODE PORT
TOP COVER PLATE
JUNCTION WITH TWISTED SECTIONS FOR TE_{20} PORT
TE_{10} SINGLE MODE PORT
BOTTOM COVER PLATE

FIGURE 6

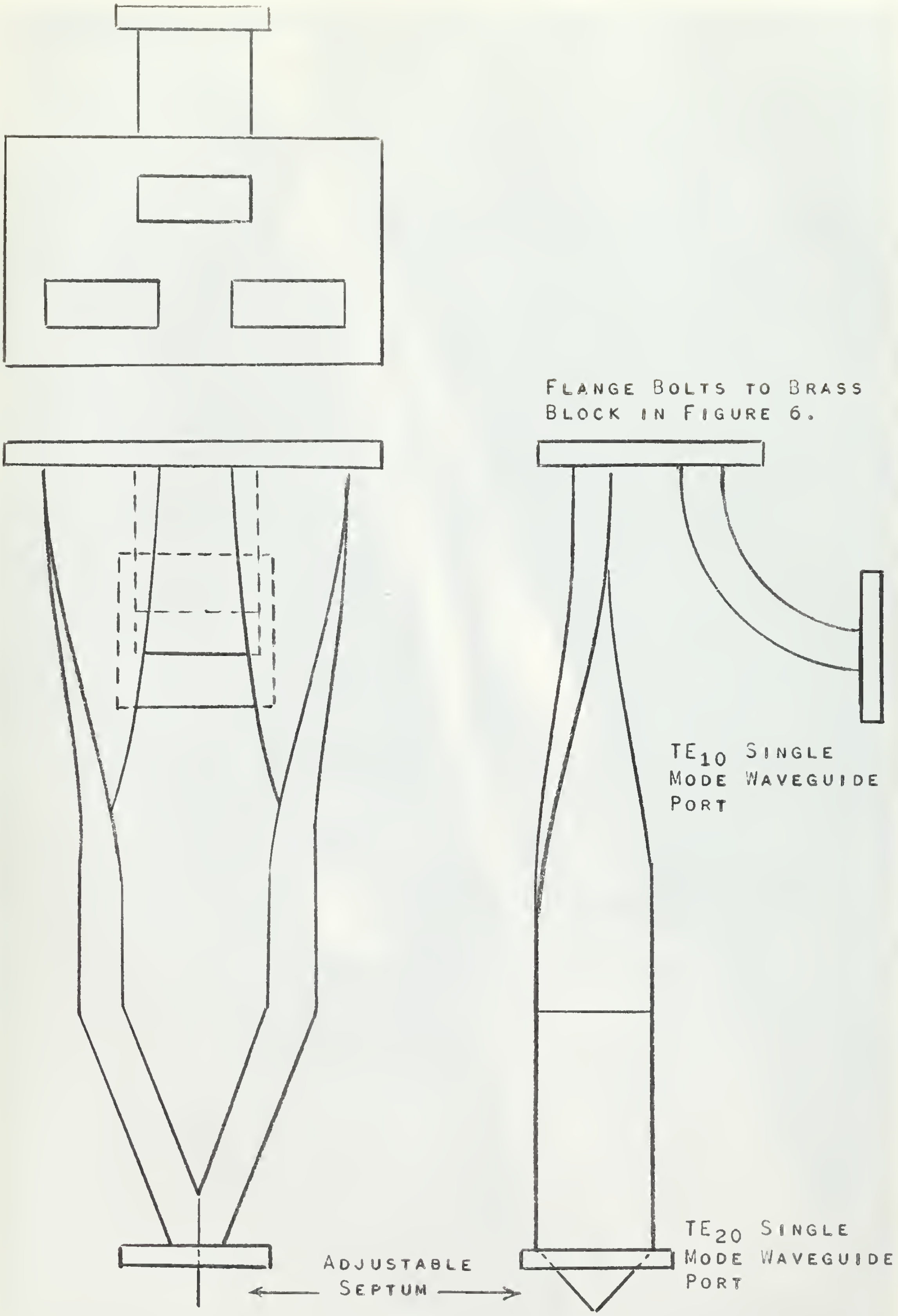

FLANGE BOLTS TO BRASS
BLOCK IN FIGURE 6.
TE_{10} SINGLE
MODE WAVEGUIDE
PORT
TE_{20} SINGLE
MODE WAVEGUIDE
PORT
ADJUSTABLE
SEPTUM

FIGURE 7

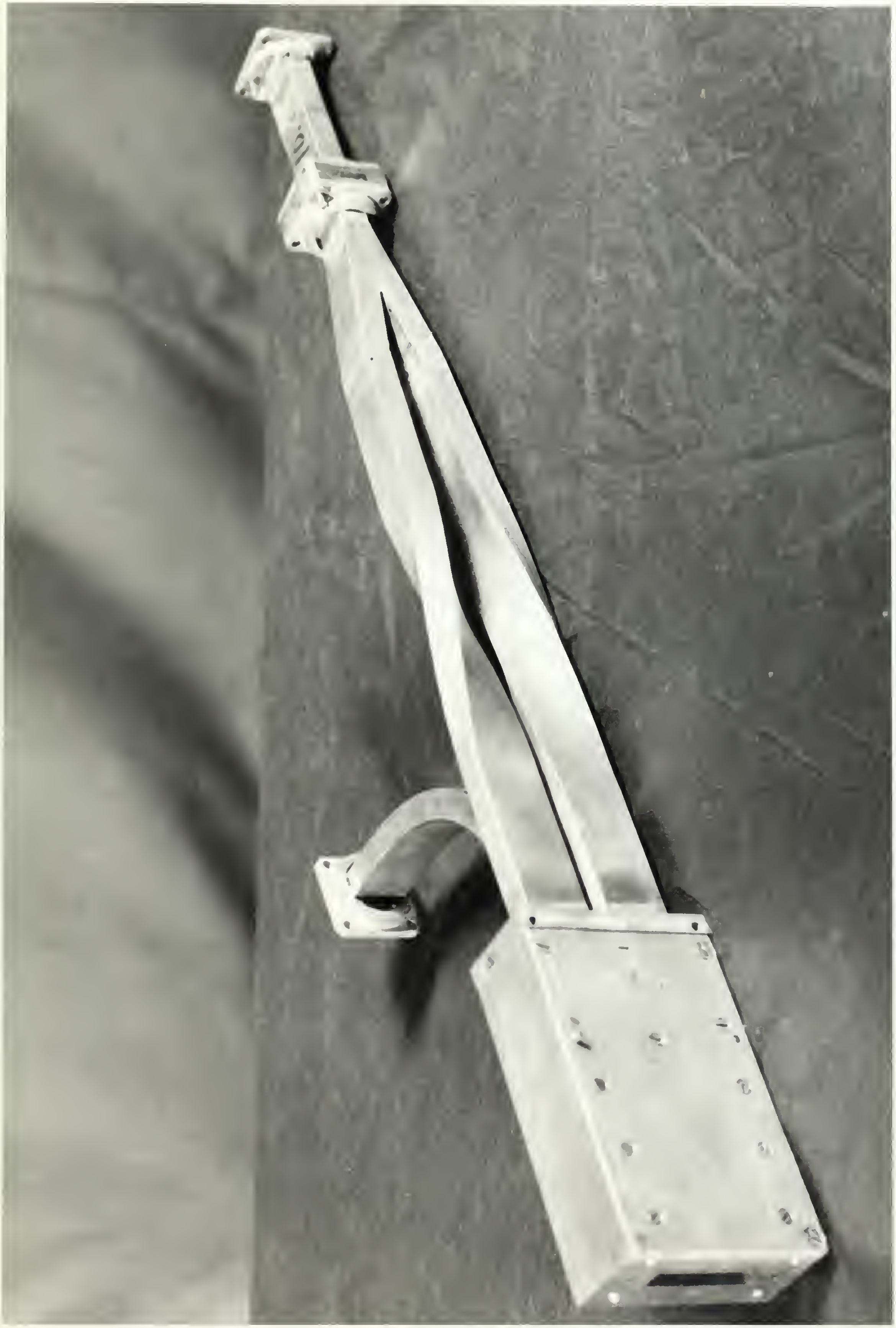


FIGURE 8

5. Measuring the Characteristics of the Mode Transducer

A ratio bridge was adapted, as shown in Figure 9, to measure the SWR at the TE_{20} and TE_{10} single mode ports and also to measure the isolation between them. This data could then be used to find the amplitudes of the scattering coefficients. (Details of the measurement bridge are covered in Footnote 1.) With approximately two watts available from the klystron amplifier the maximum sensitivity allowed measuring power ratios up to 70 db.

When the isolation between modes was first measured it was found to be on the order of 20 db. This was considerably worse than the existing mode transducers that were in use and which had an isolation of about 45 db. Adjustment of the septum at the Y junction improved isolation somewhat but the main cause of the cross-coupling between modes was the difference in electrical lengths of the twisted waveguide sections. In order to determine the potential mode separation possible with this type of mode transducer without requiring the machine shop to build an identically symmetrical device, one of the twisted sections was tuned to the same electrical length as the other by squeezing the broad faces of the waveguide together slightly. This adjustment in conjunction with the septum adjustment reduced the cross-coupling between modes to an immeasurable amount. Although excellent isolation was obtained at the frequency at which the adjustments were made (9.375 kmc) the band-

width was reduced because the phase vs. frequency characteristics of the twisted sections were now quite different.

The reflected power at the TE_{10} and TE_{20} single mode ports corresponding to $|S_{33}|$ and $|S_{11}|$ are shown in db in Figure 10. The coupling between the TE_{10} and TE_{20} single mode ports corresponding to $|S_{13}|$ is shown in Figure 11. Figures 12 and 13 show the SWR and isolation for the mode transducer previously employed. The multimode port was terminated with a multimode matched load for all measurements.

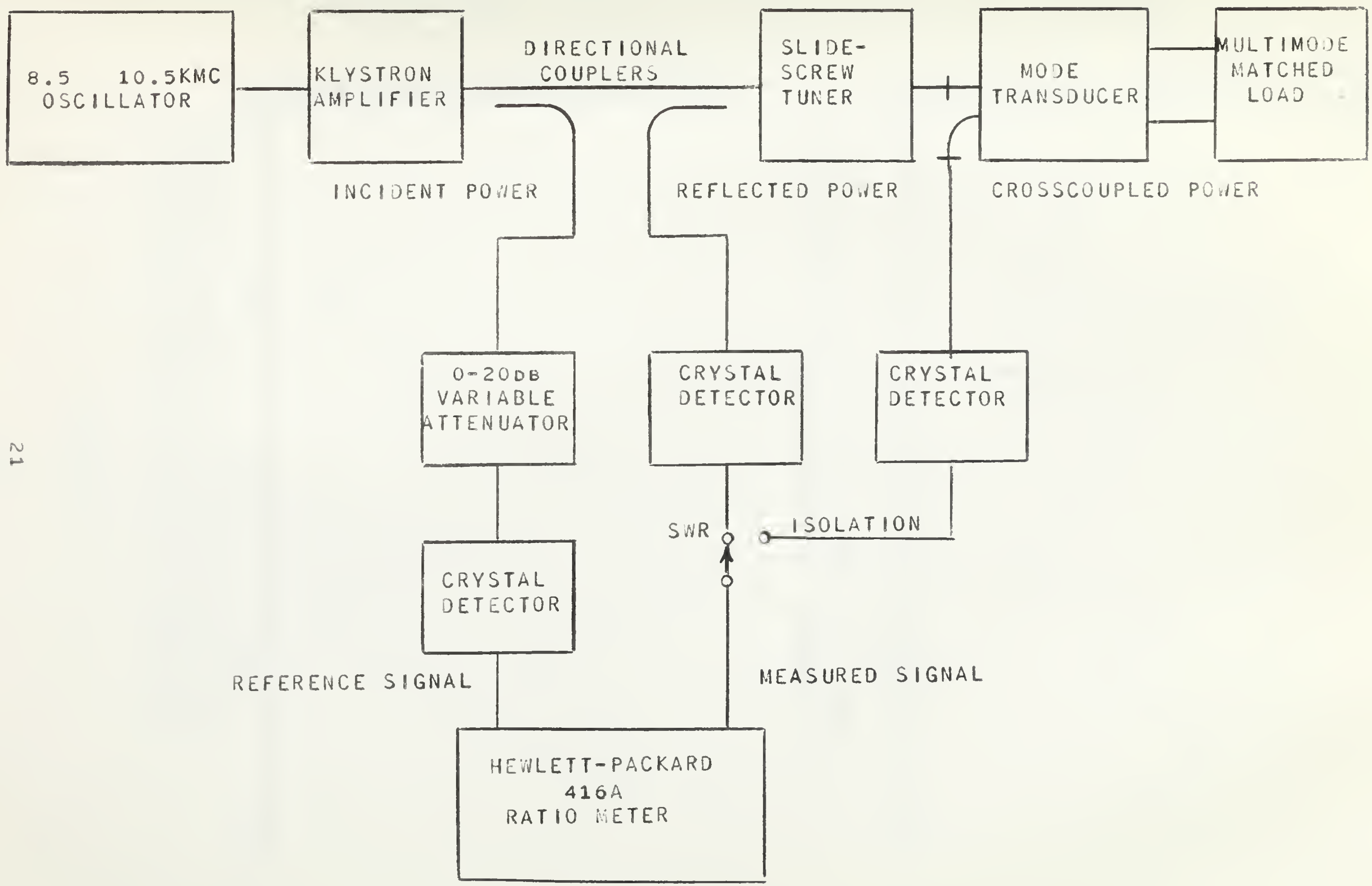
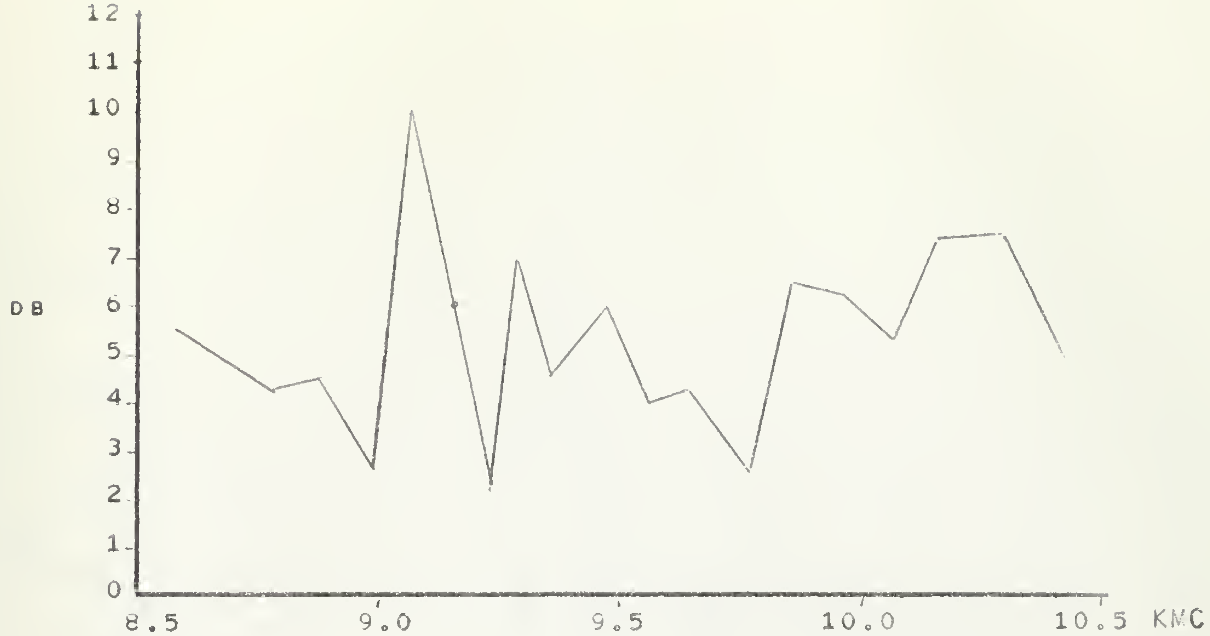
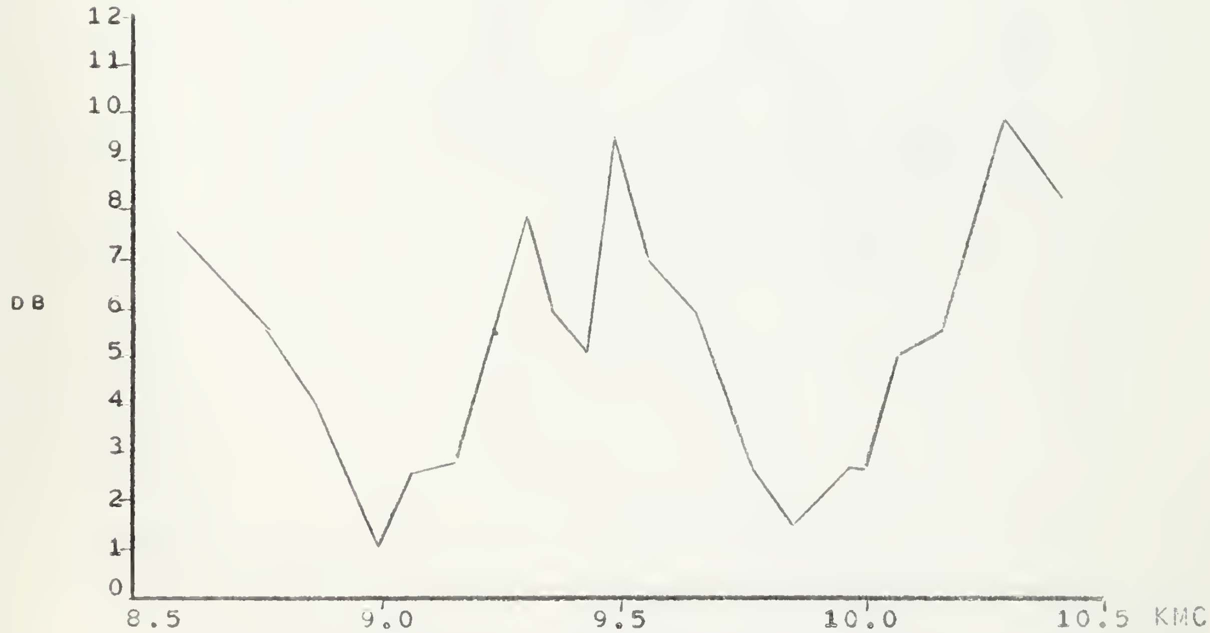

8.5 10.5KMC
OSCILLATOR
KLYSTRON
AMPLIFIER
DIRECTIONAL
COUPLERS
SLIDE-
SCREW
TUNER
MODE
TRANSDUCER
MULTIMODE
MATCHED
LOAD
INCIDENT POWER
REFLECTED POWER
CROSSCOUPLED POWER
0-20DB
VARIABLE
ATTENUATOR
CRYSTAL
DETECTOR
CRYSTAL
DETECTOR
SWR
ISOLATION
CRYSTAL
DETECTOR
REFERENCE SIGNAL
MEASURED SIGNAL
HEWLETT-PACKARD
416A
RATIO METER

FIGURE 9

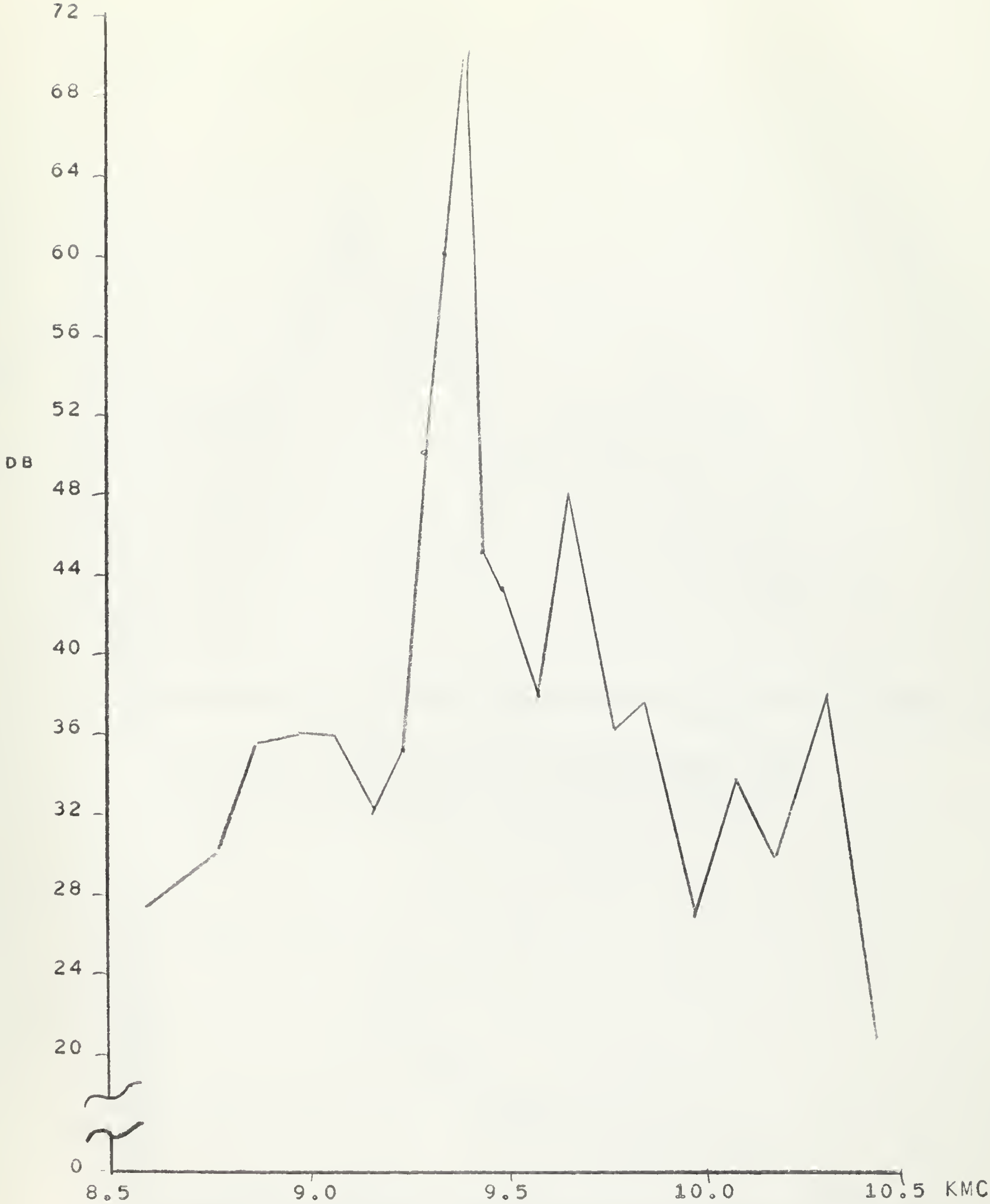


REFLECTED POWER AT TE_{10} SINGLE MODE PORT
IN DB BELOW INCIDENT POWER VS. FREQUENCY



REFLECTED POWER AT TE_{20} SINGLE MODE PORT
IN DB BELOW INCIDENT POWER VS. FREQUENCY

FIGURE 10



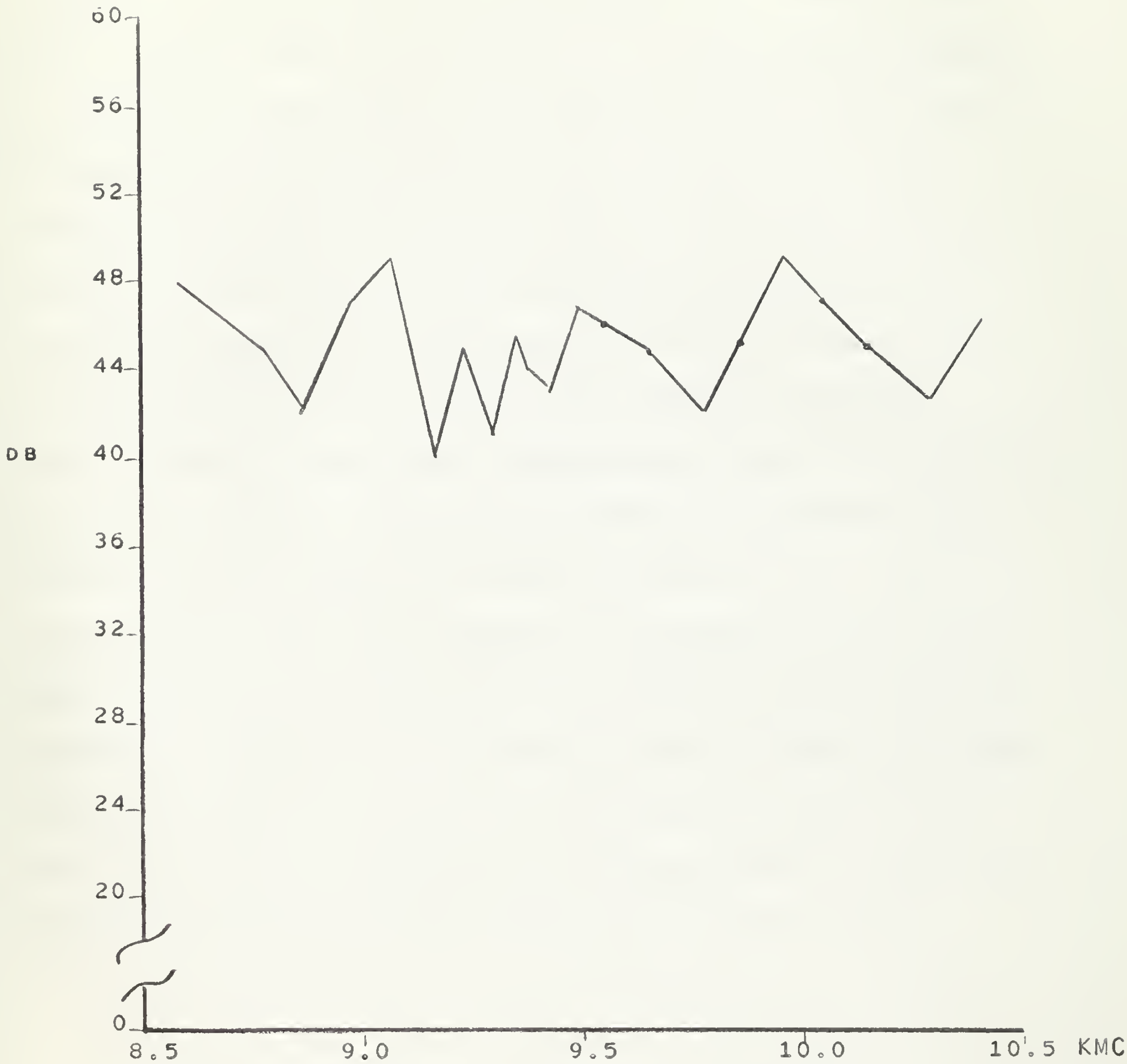
ISOLATION BETWEEN TE_{10} AND TE_{20} SINGLE MODE WAVEGUIDE PORTS VS. FREQUENCY

FIGURE 11

REFLECTED POWER AT TE_{10} SINGLE MODE PORT
IN DB BELOW INCIDENT POWER VS. FREQUENCY

REFLECTED POWER AT TE_{20} SINGLE MODE PORT
IN DB BELOW INCIDENT POWER VS. FREQUENCY.

FIGURE 12



Isolation between TE_{10} and TE_{20} single mode waveguide ports vs. frequency

Figure 13

6. Recommended Modifications to Improve Performance

The primary cause of the high SWR is the reactive load seen by (a) the TE_{20} mode incident upon the portion of the transducer designed to couple out the TE_{10}, and (b) the TE_{10} incident upon the portion of the transducer designed to couple out the TE_{20}. This problem can be reduced by extending the septum dividing the multimode guide and introducing a resistive E field vane in the center of the section through which the TE_{20} mode should propagate and similarly introduce a resistive slot in the center of the section through which the TE_{10} mode should propagate. These resistive loads would selectively absorb the rejected mode in each arm of the mode transducer eliminating the reactive terminations as a cause of reflection. These modifications would also improve the isolation since the undesired mode would be largely attenuated before reaching the region in which it is normally cancelled out. Although this would mean a 3 db loss in coupling out each of the modes, it would not create any problems when used with the multimode microwave bridge referenced in Footnote 1.

7. Conclusions

While the performance of the mode transducer was adequate for use at a single frequency in the multimode measurement bridge, the full realization of the bandwidth capabilities inherent in this design will require adherence to extremely close manufacturing tolerances. For example, at 9.375 KMC five thousandths of an inch in the X band guide corresponds to one degree of phase shift. The difference in phase shift between the two halves of the mode transducer should be held to a fraction of a degree.

Although the reflection coefficients were small enough so that slide screw tuners could be used to achieve a match, a marked improvement in the SWR should result from the modifications described in section 6.

This method of mode conversion is capable of attaining excellent isolation and a low SWR over a broad frequency range providing that the strigent requirements on physical symmetry can be met.

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